

ON THE
POLARIZATION OF LIGHT
EMITTED FROM THE SURFACES
OF
INCANDESCENT SOLIDS AND LIQUIDS.

BY
R. A. MILLIKAN, M. A.

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A STUDY OF THE POLARIZATION OF THE LIGHT EMITTED BY INCANDESCENT SOLID AND LIQUID SURFACES.

BY R. A. MILLIKAN.

I.

INTRODUCTORY.

In spite of the prodigious activity of physicists during the first three quarters of this century in attacking the problems of reflection, refraction, and polarization in all their different phases, both from the side of experiment and that of mathematical theory, the problem of polarization of light by emission seems to have received comparatively little attention. Although the fact that incandescent solids and liquids emit, at oblique angles of emergence, partially polarized light, was discovered more than seventy years ago, it does not appear even to-day to be very generally known. Few, even of the more complete text-books on Physics, make any mention of the fact. Verdet, in his "Optique," published in 1870, devotes a short paragraph to "Polarization by Emission," in which he says that "there exists upon the subject but a small number of experiments due mainly to Arago." The summary of these experiments which he subjoins, reveals none whatever which are quantitative. Since the time of Verdet, no one, so far as I am able to discover, has made any careful or elaborate study of the phenomenon with a view to ascertaining its generality, verifying or disproving Arago's assumption as to its cause, or classifying different substances with reference to their power of producing the phenomenon in greater or less degree.

Since even a hasty examination reveals the fact that different

substances emit light of widely different percentages of polarization, it appears that a study of the relations of different bodies in this respect ought either to add something to our knowledge of the optical properties of the substances considered, or else, if this particular property is deducible from the already known properties, as Arago assumed it to be, its relation to these properties ought to be definitely proved. This investigation has therefore been undertaken for the purpose, first, of making a somewhat wide range of qualitative experiments upon the nature and generality of the phenomenon; and, secondly, of subjecting Arago's explanation of the cause to the test of comparison with carefully determined experimental quantities.

II.

HISTORICAL REVIEW.

The simple facts of polarization of light by emission can best be observed, and in fact were first noticed, upon platinum. If a sheet of that metal be heated to incandescence in the flame of a Bunsen burner, and the emitted light examined by means of a Nicol prism, or any other instrument adapted to the detection of partially polarized light, it will be observed that when the experimenter is viewing the surface normally the emitted light exhibits no trace whatever of polarization, but as the instrument is inclined so as to receive rays emerging obliquely from the surface, the light begins to show evidences of polarization in a plane perpendicular to the plane defined by the normal and the emerging ray. If this plane be called the plane of emission, and the angle included between these two directions *the angle of emission*, the complete phenomenon may be roughly described by saying that the polarization increases as the angle of emission increases, and becomes, in the case of platinum, exceedingly strong as the emission angle approaches ninety degrees.

The announcement of this fact, and the consequent overthrow of the common belief that light coming immediately from self luminous bodies is always natural, was first made in 1824 by Arago. In a report made in that year to the Royal Academy of Sciences (see *Annales de Chimie et de Physique* (1) 27, p. 89.) he announced that he had some time before made a series of experiments upon the light which emanates from incandescent bodies. "He found that if the bodies are solid or liquid this light is partially polarized by refraction when the rays observed form with the emitting surface an angle of a small number of degrees. As for the light of an ignited gas it presented under no inclination traces of sensible polarization." From these ex-

periments he drew the conclusion "that a considerable portion of the light which enables us to see incandescent bodies is produced in the interior and at depths which are not yet completely determined." "Even when the surface of a solid or liquid was not well polished," Arago still found that he "was able to detect evident traces of polarization." The substances upon which he experimented and from the observation of which he drew his conclusion were only four in number, viz.—solids, wrought iron and platinum; liquids, molten iron and glass (See *Astronomie Populaire* II., p. 103.). He made *no* quantitative measurements, nor even used an instrument which was capable of indicating roughly *amounts* of polarization. His polariscope consisted of a single quartz crystal cut perpendicularly to the optical axis and a crystal of Iceland spar. The latter produced a double image of an opening in a diaphragm placed just beyond the crystal of quartz. The two images were of course colored when the light was polarized and uncolored when it was natural.

Arago applied the results of his experiment to the determination of the character of the sun's surface. Being unable to detect any trace of polarization in the light emitted by the outer edge of the sun's disk, he drew the well-known conclusion that the surface of the sun can be neither liquid nor solid, but must be gaseous.

After the discovery of the polarization of heat, and the construction of an instrument by Melloni for its detection and measurement, Provostaye and Desain examined the heat rays emitted by luminous platinum and found that they, like the light rays, were polarized in the plane perpendicular to the plane of emergence. Their experiments were few in number and confined entirely to platinum. In 1866 Magnus extended this method of experiment to obscure heat rays, making quantitative measurements upon the heat emitted at the temperature of 100° C, and at an angle of 35° . His experiments embraced the following list of substances: Paraffine, glycerine, white wax, melted calophony, rubol, black glass, transparent glass, quicksilver, aluminium, copper and tin. For these substances he found a polarization at 35° ranging from 5% to 27%. He drew the conclusion that obscure heat, like light, must undergo refraction in emerging from the surface of the radiating body.

Verdet, in the paragraph upon polarization by emission previously referred to, while stating that little has been done in the investigation of the subject, gives the same explanation of the phenomenon as that first offered by Arago. He says that "it is due to the fact that it is not alone the surface molecules which radiate light; those of the interior layers also radiate, at least

to a certain depth; and the rays emitted by the interior molecules undergo refraction at the surface." Since the time of Verdet, I believe no one has made any experiments upon the subject except Violle, who has a brief note in the *Compte Rendu* of 1887, Vol. 105, p. 111, in which he states that, while making some other experiments upon molten silver, he took occasion to measure the percentages of polarization in the light emitted by that substance at various incidences. He plotted the curve of these percentages and found that it was very well represented by the empirical formula $p_e = (1 - \cos i) (1 + \cos 75^\circ + \frac{1}{5})$, where p_e represents the ratio of polarized light to the whole light in the emitted beam and i the angle of incidence.

Assuming, then, the phenomenon to be due to refraction, he argues that the equality of the amounts of polarization in the reflected and refracted beams would require that $e p_e = r p_r$, where e is the proportion of the whole light emitted, r the proportion reflected, p_e the proportion of polarized light in the emitted beam, and p_r the proportion of polarized light in the reflected beam. Then, since the whole light is either emitted or reflected, $e + r = 1$, and the formula $r = \frac{pe}{pe+pr}$ immediately follows. Taking the experimental values which have been determined for p_r by reflection at ordinary temperatures, he finds that his own results for p_e , when substituted in this formula, give a uniformly high reflecting power for molten silver; a result which agrees with the known properties of ordinary polished silver. This forms the nearest approach to a verification of Arago's assumption which has yet been given.

Such is the extent of the work which has thus far been done upon polarization by emission.

III.

DISCUSSION OF ARAGO'S EXPLANATION.

The explanation of Arago and Verdet is as yet the only one which has been offered to account for the phenomenon. This explanation does not rest upon careful experimental proof, and, furthermore, there seems to be considerable reason for doubting its correctness. According to that explanation the light which comes to the eye from the *surface* particles is *natural* light; but mixed with this unpolarized light is a quantity of light which has worked its way up from uncertain depths, has undergone reflection and refraction at the surface, and is consequently polarized upon emergence. Aside from the intrinsic difficulty of this conception, the first experiments which were made in this

research upon platinum seemed to be inconsistent with such an explanation; for, when a well polished platinum strip was heated to incandescence by means of an electric current and the glowing surface examined by means of a double Wollaston prism, the polarization was found to be so nearly complete for angles in the neighborhood of grazing emergence that one of the images almost disappeared. But, since platinum is known to be altogether opaque, except in the case of exceedingly thin laminae, it would seem as though the surface molecules must play a considerable part in the luminosity of the glowing metal; so that, even if the assumption were made that the laws of reflection and refraction would require complete extinction of the ray polarized parallel to the plane of emergence, there still ought to be a considerable amount of light emitted in this plane from the surface molecules; at least, a sufficient quantity to prevent so nearly complete extinction as experiment showed to exist for angles of 88 or 89 degrees.

The only apparent method of reconciling the facts with Arago's explanation was to assume that the opacity of the platinum was greatly diminished by an increase in its temperature. And yet, such experiments as were made to determine whether or not this was the case, gave only negative results. The thinnest sheet of platinum which was capable of being heated to incandescence without melting, was placed in the focus of a powerful beam of light from an arc lantern, the beam having been first polarized by transmission through a Nicol. The plane of the glowing platinum being perpendicular to the beam, the light emerging normally on the other side of the platinum was examined by means of a delicate polariscope. No trace of polarization was detected. Neither could the outlines of the focus be distinguished on the side of the platinum away from the lantern. The sheet of platinum employed was evidently just as opaque as at a lower temperature.

This difficulty of accounting for the extreme polarization noticed at large angles of emergence appeared to be considerably diminished if another cause for the phenomenon were assumed than that given by Arago.

According to the conclusions of Fresnel, Cauchy, Stokes, Mascart and most of the advocates of the elastic solid theory of light, the direction of vibration of the ether particles in plane polarized light is perpendicular to the plane of polarization. It would follow that the light emitted at large angles by platinum vibrates mainly in the direction of the normal to the surface. It is not unnatural to suppose that at the boundary between very dense and very rare media, like platinum and

air, there may be less resistance to vibration in a direction away from the surface than in a direction parallel to the surface, and therefore that the light emitted is composed mainly of vibrations in a direction normal to the surface. If this were the case the light emitted normally would be unpolarized, while that emitted at oblique angles would be polarized in the plane perpendicular to the plane of emission. Furthermore, the polarization would increase with the angle and might be very great at large angles, in case the difference in density between the two media were very great—conclusions all of which are in accordance with the facts. In view, then, of the inability to account, by Arago's assumption, for the extreme polarization at large angles of emergence, and in view of the plausibility of the other explanation, the following qualitative experiments were made in order to determine with more certainty the nature of the phenomenon.

IV.

QUALITATIVE EXPERIMENTS.

The object of this part of the research was :

(1) To make certain that the property of polarization is due to the incandescent body itself, and is not caused by the refraction of the light as it passes through the layers of air of varying density which rest upon the luminous surface; and,

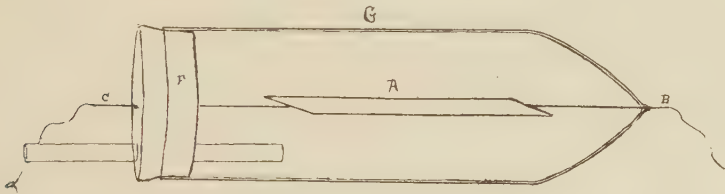
(2) To make observations upon as wide a range of substances as could be made to emit light without combustion, in order to ascertain whether any substance could be found which does not possess the characteristic, and also in order to determine in a general way the relations of different bodies with reference to this property.

For these purposes two instruments were employed; the first, a polariscope similar to that of Arago, save that the simple quartz crystal was replaced by a bi-quartz plate, and the crystal of calc-spar by a double Wollaston prism. This is the same instrument which was afterwards used by Arago in his polarimeter, and it has an advantage over the first form in that the two colors to be compared are brought into immediate juxtaposition. It is delicate to the extent of detecting a polarization of about 3% (as was shown by succeeding experiments), when white light is under examination. When the light to be tested is monochromatic, as was the case in some of the following experiments, the second form of polariscope was found to be preferable.

In this instrument the bi-quartz is replaced by a cube of glass which has been subjected to strain in cooling. A Nicol

also takes the place of the Wollaston prism of the first polariscope. The glass, being in a state of strain, is doubly refracting and exhibits with polarized light the familiar dark or light cross which is characteristic of doubly refracting crystals, when cut perpendicularly to the optic axis and viewed by convergent light. With this instrument a polarization of two or three per cent. could be easily detected, and it had the further advantage of indicating immediately the azimuth of the plane of polarization. Also, by careful observation of the distinctness of different parts of the figure, it was possible, after a small amount of practice, to estimate with considerable correctness the *degree* of polarization.

1. In all experiments which have been previously performed upon this subject, the white-hot body has been in immediate contact with the air. The emitted light was therefore obliged to pass through layers of air of varying density before it reached the eye of the observer. That the light might not thus suffer



a large number of refractions between the incandescent body and the eye, and so be endowed with the property in question, seemed entirely possible. It was therefore necessary to make some experiment in order to determine whether or not this was the entire or partial cause of the phenomenon. For this purpose the contrivance shown in the diagram was employed. A strip of platinum foil A about four cm. in length and five mm. in width was attached to the platinum and copper wires B and C. The former was sealed into the glass tube G, and the latter was passed through the cork F which closed the other end of the tube.

The instrument was first sealed with wax and then connected with the air pump by means of the small tube D, and with a strong electric current by means of the wires B and C. Care was taken to place the platinum strip as near the axis of the tube as possible, in order that light emitted by it might pass normally through the sides of the tube. Otherwise polarization

would have been caused by the passage of the beam through the glass itself. The tube being exhausted until the gauge showed a pressure of only four millimeters, the current was turned on and the glowing strip examined by means of the bi-quartz polariscope. The emitted light was still found to be polarized for oblique angles of emergence, and did not appear to have undergone any change in intensity. In order, however, to ascertain whether or not the effect of the air was altogether negligible, more delicate experiments were necessary. These will be hereafter described.

It may also be added at this point that experiments were made upon the glowing platinum by means of a Jamin compensator, in order to make certain that the light under consideration was simple partially polarized light, and not elliptically polarized. From our knowledge of the way in which elliptically polarized light is produced, viz., by causing retardation or acceleration in one of the components of plane polarized light, it would immediately be inferred that the light emitted by the platinum is not elliptically polarized, since there is no reason to suppose that it was at first *plane* polarized. The correctness of this conclusion was immediately shown by the use of the compensator. For, the compensator having been set at zero by the use of plane polarized light, when the beam of light from the platinum was made to fall upon the instrument, the dark central line showed no displacement whatever from the zero position. Had the light been elliptically polarized the black central line would have been of necessity somewhat displaced, no matter what angles the compensator planes made with the rectangular axes of the ellipse. Furthermore, for elliptically polarized light there is always some position of the analyser which gives an absolutely black line, whereas, if the light is partially polarized, the line can never be any darker than the field which is illuminated by that quantity of natural light which is contained in the partially polarized beam. Applying these considerations to the light emitted by the platinum it was shown without question that the light is partially and not elliptically polarized.

2. Having thus examined the nature of the light, and proved that the phenomenon is inherent in the body itself, experiments were made upon the following substances with results as indicated.

SOLIDS.—*Metallic.*

Platinum (polished).—Polarization very strong near grazing emergence, but falling off rapidly as the angle diminishes. Scarcely perceptible at ten degrees.

Silver.—Polarization strong, larger for small incidences than in the case of platinum.

Gold.—Polarization strong ; similar to platinum, but apparently less for large angles.

Copper.—Polarization weak, probably due to roughening of surface through oxidization.

Brass.—Polarization weak—(oxidization).

Iron.—Polarization weak—(oxidization).

SOLIDS.—*Non-metallic—transparent.*

Glass.—Polarization weak ; imperceptible except at large angles of emergence.

Mica.—Polarization weaker than in glass. Surface roughened by heat.

SOLIDS.—*Non-metallic—opaque.*

Porcelain.—Polarization similar to that produced by glass.

Black Glass.—Polarization similar to that produced by transparent glass.

LIQUIDS.

Molten Silver } Polarization similar to that in solid state.
 “ Gold }

“ Iron.—Polarization strong ; almost as strong as in molten gold.

“ Bronze.—Polarization strong ; almost as strong as in molten gold.

Lead.—Polarization weaker than for preceding metals. (Difficult to get a clear surface).

Zinc.—Polarization weaker than for preceding metals.

From these experiments it will be seen, 1st, that the metals show uniformly high percentages of polarization so long as the surface is non-diffusing ; 2nd, that none of the non-metallic substances used produce strong polarization at any angle ; 3rd, that the transparency or opacity of a substance has apparently little effect upon its power of producing polarization in the emitted light ; and, 4th, that any cause which interferes with the perfect smoothness and regularity of the surface destroys in large measure the polarization.

V.

INSTRUMENT EMPLOYED FOR QUANTITATIVE EXPERIMENTS.

In order to accomplish the second and main object of the research, it became necessary to devise some means of making accurate determinations of the relations of the *constants* of the partially polarized beam. In case of elliptic polarization the constants are best determined by means of a Babinet compensator, but this instrument fails entirely for partially polarized light. The instrument which has been most employed for such work by previous investigators is the Polarimeter of Arago.

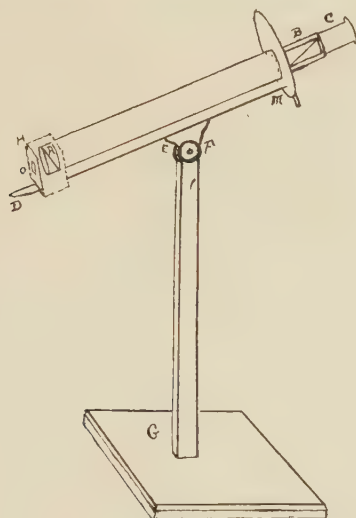
This is an instrument simple enough in principle, but difficult in construction. Moreover it does not possess a very high degree of accuracy, owing to the fact that its use depends upon the detection, by means of a bi-quartz polariscope, of the exact point at which all polarization disappears from a beam of light. The beam of light to be examined for polarization is made to pass through a pile of glass plates set in such a manner with reference to the beam as to produce, by refraction, polarization in a plane at right angles to the plane in which the beam is already partially polarized. The inclination of the plates is altered until the halves of the bi-quartz plate show no traces of color. The instrument having been previously graduated by means of a beam of known polarization, the amount of polarization in the beam in question can be immediately read off upon the graduated circle.

The graduation is effected by passing a plane polarized beam through a thick crystal of Iceland spar. Then, by the Law of Malus, the polarization in the transmitted beam is given by the cosine of twice the angle included between the plane of polarization of the incident beam and the principal plane of the crystal.

Both because of the difficulty of construction of this instrument, and because my own experiments with the bi-quartz polariscope made me distrustful of the accuracy with which the point of no polarization could be determined, another form of instrument was devised for these experiments which is greatly superior to the Arago polarimeter in simplicity, and is probably more than equal to it in accuracy. The instrument had been constructed and used for several months before the discovery was made that the credit of the first conception and use of this method of measuring the constants of partially polarized light is due to Cornu.

In view of the exceeding naturalness and simplicity, as well as the accuracy of the method, it is surprising that it was not earlier discovered and has not been more generally employed. Cornu's description of his instrument was published in '82 in the "*Ass'n Francaise pour L'Avancement des Sciences, Compete Rendu*;" but so far as I can discover, no reference was made to it at the time in any of the scientific journals, nor has it taken its place among other polarimeters in any of the text books on optics. The instrument as constructed and used for the purposes of these experiments was as follows. A rectangular opening O, 1 mm. in width and 2.5 mm. in length, was made in a diaphragm H, which stood a short distance in front of the double Wollaston prism A. The prism was rotated until the extraordinary image of the opening was to the left of the ordinary; the distance of

the screen from the prism was then adjusted until the opposite edges of the two images exactly coincided. A Nicol prism B, capable of rotating about its axis and furnished with a graduated circle and vernier for reading azimuth to a tenth of a degree, constituted the only other essential part of the instrument. A small telescope C was used for viewing the images of the rectangular opening O. The instrument was mounted upon a support G, furnished with a horizontal axis at F, about which the upper portion of the apparatus could be revolved. The axis of the tube bearing the double prism and Nicol could thus be in-



clined so as to make any desired angle with the vertical. Since the two images furnished by the double prism consist of light polarized in planes at right angles to each other, the rotation of the Nicol will evidently extinguish each of them in turn. There will be four extinctions in the course of a complete revolution of the Nicol, and between any two extinctions there is a point for which the images, as seen through the Nicol, have exactly equal intensities. If now a partially polarized beam is under examination, and if, the plane of polarization of this beam being known, the principal sections of the prism are set parallel and perpendicular to this plane, that position of the Nicol which equalizes the two images, evidently defines the relation between

their original intensities, which is also the relation between the constants of the partially polarized beam.

If we let a and b represent the original amplitudes of vibration in the two images, then the intensities of these images are represented by a^2 and b^2 respectively. The proportion of polarization is evidently the difference between these intensities divided by their sum. If w is the angle which the transmitting plane of the Nicol makes with the direction of vibration of the more intense of the two beams, say a^2 , then the intensities of the two images as seen through the Nicol will be, by the law of Malus,

$$a^2 \cos^2 w \text{ and } b^2 \sin^2 w. (1).$$

Hence for the position of equality we have,

$$a^2 \cos^2 w = b^2 \sin^2 w. (2),$$

$$\text{or } \frac{a^2}{b^2} = \frac{\sin^2 w}{\cos^2 w} (3)$$

If we call the degree of polarization in the original beam p , we have

$$p = \frac{a^2 - b^2}{a^2 + b^2} (4) \text{ or, from } (3)$$

$$p = \frac{\sin^2 w - \cos^2 w}{\sin^2 w + \cos^2 w} = - \frac{\cos^2 w - \sin^2 w}{1} = - \cos 2 w. (5)$$

Hence, when the position of the Nicol which produces equality in the images has been found, the amount of polarization is immediately given by (5).

Cornu, in discussing the instrument, shows in addition, that, when the principal sections of the partially polarized beam are not known, the degree of polarization may still be found by taking one set of readings in any position whatever of the axes of the double prism, and then rotating the whole instrument through an angle of 90° and taking a second set of readings. The degree of polarization can then easily be shown to be given by the formula

$$p = \sin (w_2 - w_1)$$

In this work, however, we are not concerned with this last formula, since the principal sections were always known.

VI.

ADJUSTMENT OF THE INSTRUMENT.

Since the series of experiments here considered were all made upon horizontal surfaces, and since the polarization of the emitted light is always in a plane normal to the surface, but one adjustment of the instrument was necessary, viz., that of bring-

ing the principal sections of the double prism unto coincidence with the horizontal and vertical directions.

This adjustment was effected in the following way. The axis of the tube bearing the prism and Nicol was first set, by measurement, parallel to the cross-piece D M of the supporting frame. The base G was then carefully levelled by means of a common level. The levelling of D M then brought the axis of the tube into coincidence with the horizontal line. The telescope was then focussed upon a very fine line of light reflected from the edge of a carefully levelled sheet of white paper placed just in front of the rectangular opening O. Since the line joining the two images produced by a crystal of calc-spar is always paral-
lel to the optical axis of the crystal, it follows, that, when the two images of the horizontal line of light form with each other an unbroken line, the principal sections of the crystal have the desired directions, and the adjustment is perfect. The line of light used in this case was so narrow that the adjustment was able to be made with great accuracy. This done, the crystal was permanently fastened in position. Thereafter, it was only necessary, before each observation, to level the base of the instrument G, in order to bring the principal sections of the crystal into the desired positions.

In order to set the axis of the tube at any desired angle with the vertical, the cross-piece D M, parallel to this axis, was levelled, and the angle D E G, between the movable arm and the vertical support, was measured by means of a protractor. The zero position being thus determined, any desired inclination could be secured by giving to the angle D E G the proper value.

VII.

DEGREE OF ACCURACY OF THE INSTRUMENT.

The great sensitiveness of the eye in detecting slight differences in the intensities of images of the same color, when brought into close proximity, has often been the subject of remark. Cornu claims that the position of equality can be determined with a precision that reaches $\frac{1}{10}$ of a degree. My own observations would not lead me to attribute to the instrument so high a degree of accuracy. Furthermore these observations are subject to the objection which attaches to all photo-metric experiments, that the sensitiveness of the eye varies greatly with the physical and mental condition of the observer. At times the extreme difference in my readings for a given set of conditions would be as high as $2\frac{1}{2}$ degrees. Usually, however, the extreme difference was not more than $1\frac{1}{2}$ degrees. For the sake of testing the

probable accuracy of the results which are to be given later, several sets of observations were made upon the unpolarized light of a gas flame. The following illustrate about the average course of the readings. The zero of the instrument not being known, the positions of equality on each side of the positions of extinction were determined :

Left.	Right.
50.5	39.2
50.0	39.1
50.0	38.7
50.4	38.8
51.3	40.0
50.0	40.0
51.0	40.0
51.1	39.9
50.2	39.2
49.5	39.0
50.4	39.37
$2 w = 89.77$	
$w = 44.88$	

Since the light from a gas flame is unpolarized, the value of w should have been 45° . The difference is not large, but is slightly greater than the maximum error ascribed to the instrument by Cornu. The above is about an average set of readings. The extreme difference is 1.8° , a difference perhaps slightly greater than that usually found.

A second slight error may sometimes arise in the use of this instrument from the fact that the two images produced by the double prism do not correspond to exactly the same points on the luminous surface. Hence, in order that the results may be correct, it is necessary that the adjoining portions of the incandescent surface be exactly alike. In none of the experiments here recorded were the portions of the luminous surface producing the two images more than 3 mm. apart. Care was always taken to direct the instrument toward a portion of the surface which appeared to be entirely uniform. This error may, I think, be safely disregarded in all of the following cases except one, which will be mentioned later.

A third remark which should be made upon the accuracy of the instrument is that observations for large amounts of polarization are less subject to error than those made upon small amounts. For, since the intensities of the two images compared are pro-

portional to $\sin^2 w$ and $\cos^2 w$, the change in intensity of one of them will be very rapid when w is in the neighborhood either of zero or of ninety degrees. When, however, w is near 45° , the change in intensity corresponding to a small change of angle is comparatively small. Hence, when the polarization is large, and w consequently either large or small, the position of equality can be determined with considerably greater accuracy than when the polarization is weak and w in the vicinity of 45° .

The results obtained for large angles may therefore be considered more trustworthy than the results for small angles.

VIII.

MEASUREMENT OF THE AIR EFFECT.

In the qualitative experiments previously described it was ascertained that the amount of polarization was at least not greatly affected by the contact of the air with the heated surface. Before proceeding to careful quantitative measurements it was necessary to determine whether or not its effect upon the phenomenon is altogether negligible. This could be easily done by means of the polarimeter.

The sealed glass tube containing the platinum strip was again connected with the air-pump, and the air exhausted until the pressure was about 4 mm. The current was turned on, the polarimeter arranged so as to receive the light emitted from the glowing surface at an angle of about 80° , and the Nicol turned until the images were brought into equality. The stop cock was then suddenly turned and the air admitted. No change whatever could be perceived in the equality of the images. The experiment was repeated a number of times and in a variety of ways, but always with the same result. The conclusion was, that, if the air has any effect whatever upon the proportion of polarization in the beam, that effect is so slight as to be ALTOGETHER NEGLIGIBLE; a result exceedingly fortunate for the purposes of this investigation, since, had it been necessary to work upon substances in a vacuum, the following experiments would have been much more difficult, if not altogether impossible.

IX.

EXPERIMENTS UPON URANIUM GLASS.

The chief difficulties which beset the investigation of Polarization by emission are, 1st, the difficulty of obtaining a perfectly *definite* and *regular* incandescent surface with which to work; and 2nd, the difficulty of ascertaining with certainty the

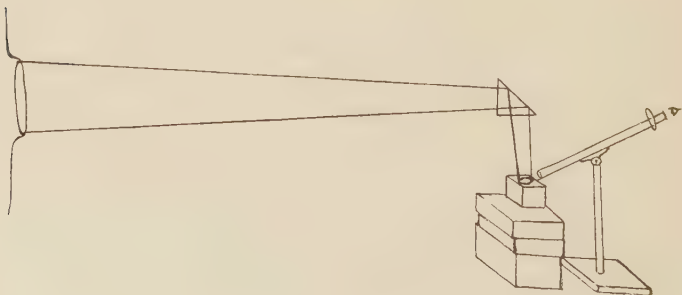
optical constants of *any* bodies at the temperature of incandescence.

The similarity between a body emitting light by incandescence and a body emitting light by fluorescence was first suggested to me by Professor Rood. According to Tait the phenomenon of fluorescence is confined mainly to the surface layers. Whatever the cause, then, of polarization by emission, the light coming from a fluorescent surface ought to be polarized in the same way as the light coming from glowing platinum.

Experiment showed this conclusion to be entirely correct. The polarization seen in the uranium glass was similar in every respect to that observed in incandescent porcelain, being scarcely discernible at any angle less than 50° , but becoming quite marked between 85° and 90° , and evidently reaching a maximum at grazing emergence.

That this polarization was not due to diffusing particles on the surface was certain for three reasons. 1st. The surface was NOT a diffusing surface except to an exceedingly small extent. 2d. The light which exhibited the phenomenon of polarization was the characteristic yellowish-green light which uranium emits, and not the blue light which fell upon the surface. 3d. The reflecting particles on the surface would have produced a polarization *in* the diffusing plane, *i. e.*, in the plane defined by the direction of the beam which entered the instrument and the direction of the incident beam, which was in this case normal to the surface. As a matter of fact the polarization was in a plane perpendicular to this plane.

Here, then, was an instance of polarization by emission in which the surface was perfectly definite and at the same time the optical constants of the substance could be easily and accurately determined.



Accordingly a careful series of observations was made with the polarimeter. The experiments were all conducted in a well darkened room, and care was taken not to allow any light to enter the instrument except that emitted by the uranium glass. In order to make the determination of the angles of emission convenient, the light from the lantern was thrown vertically down upon the surface of the uranium glass by means of total reflection in a right angled prism. The cube of glass was carefully levelled so that the emitting surface was always horizontal. The arrangement of apparatus is shown in the figure.

Ten readings were taken for every angle of emergence. The results are given in full.

87.5°		85°		80°	
Left.	Right.	Left.	Right.	Left.	Right.
40.5	29.8	42.3	30.7	43.0	32.0
39.5	29.9	43.0	31.2	43.3	32.5
39.8	28.5	42.5	30.5	43.8	32.3
40.0	29.0	41.7	30.9	44.0	32.6
39.0	28.9	40.9	31.0	43.3	32.3
39.76	29.22	42.1	30.86	43.48	32.34
$2 w = 68.98^\circ$		$2 w = 72.86^\circ$		$2 w = 75.82^\circ$	
$p = .358$		$p = .293$		$p = .245$	

75°		70°		65°		50°	
Left.	Right.	Left.	Right.	Left.	Right.	Left.	Right.
44.5	34.5	45.5	35.0	47.0	37	49.1	38.2
45.0	35.0	46.4	36.4	47.2	36.3	48.9	38.2
44.3	35.0	46.5	36.5	47.5	37.5	49.0	38.9
44.5	33.5	47.0	35.0	48.0	37	50.0	39.3
44.5	34.4	46.3	36.2	47.4	36	49.3	39.0
44.56	34.5	46.34	35.8	47.62	36.8	49.26	38.52
$2 w = 79.06^\circ$		$2 w = 82.1^\circ$		$2 w = 84.4^\circ$		$2 w = 87.78^\circ$	
$p = .191$		$p = .139$		$p = .098$		$p = .039$	

The chief difficulty encountered in making these determinations was the lack of perfect uniformity in the emitting surface. The uranium glass, being rendered selfluminous by the beam from the lantern, could not have entire uniformity over its surface unless the illuminating beam was uniform in intensity, which was not the case. The images corresponded to points on the surface not more than 2 mm. apart, and yet it was found that the equality of the images could be sometimes disturbed by directing the instrument toward a new portion of the field. As great care as possible was taken to direct the polarimeter toward such por.

tions of the field as appeared to have a uniform illumination, and it is not thought that the error due to this cause could have been great.

Phosphorescent bodies were also examined for polarization, but the light emitted by such bodies is so weak that no definite results were obtained.

X.

APPLICATION OF FRESNEL'S FORMULÆ FOR VITREOUS REFLECTION.

The main object of this research being to determine whether or not polarization by emission could be experimentally proven to be a phenomenon of refraction, Fresnel's laws for reflection and refraction, which have been shown by many experiments to accurately represent the facts, were now applied to the determination of the amounts of polarization which should be produced by single refraction of light passing through the boundary surface between uranium glass and air. In order to apply these laws it is necessary to assume that all of the light emitted by the uranium glass, whether coming *from the surface molecules or from the interior layers*, has undergone the process of refraction—an assumption not contained in Arago's explanation of the cause of the phenomenon.

Taking the intensity of the incident ray as unity, Fresnel's formulæ give for the intensities of the reflected and refracted rays, when the incident beam is plane polarized in the plane of incidence,

$$\begin{aligned}\text{reflected ray} = r_1 &= \frac{\sin^2(a - \beta)}{\sin^2(a + \beta)} \\ \text{refracted ray} = r_1' &= \frac{4 \cos^2 a \sin^2 \beta}{\sin^2(a + \beta)}\end{aligned}$$

a being the angle of incidence and β the angle of refraction.

For a ray plane polarized in a plane perpendicular to the plane of incidence

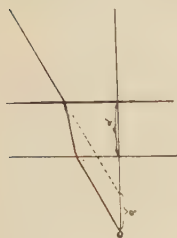
$$\begin{aligned}\text{reflected ray} = r_2 &= \frac{\tan^2(a - \beta)}{\tan^2(a + \beta)} \\ \text{refracted ray} = r_2' &= \frac{4 \cos^2 a \sin^2 \beta}{\sin^2(a + \beta) \cos^2(a - \beta)}\end{aligned}$$

Since ordinary light may be considered as composed of two equal plane polarized beams, polarized in planes at right angles

to each other, the amount of polarization in a beam of natural light which has undergone single refraction is

$$\frac{\frac{1}{2} r_2' - \frac{1}{2} r_1'}{\frac{1}{2} r_2' + \frac{1}{2} r_1'} = \frac{r_2' - r_1'}{r_2' + r_1'} = \frac{\frac{1}{\cos^2(a - \beta)} - 1}{\frac{1}{\cos^2(a + \beta)} + 1} = \frac{1 - \cos^2(a - \beta)}{1 + \cos^2(a + \beta)} = p$$

The only unknown quantity in this formula is the angle β . In order to determine β for any given angle it was only necessary to determine the index of refraction of the uranium glass.



The glass being of considerable thickness the microscope method was the one best adapted to this determination. d being the thickness of the glass, and a the change in focus due to the introduction of the glass between the object o and the objective, the index u is given by the formula (see Kohlrausch Praktische Physik, p. 151).

$$u = \frac{d}{d-a} = \frac{326 \text{ mm}}{326 \text{ mm} - 110 \text{ mm}} = 1.51 = \frac{\sin a}{\sin \beta}$$

The substitution of the various values of β , thus found, in the formula for p gave the following values:

a	β	p
$87^\circ 30'$	$41^\circ 25'$	351
85°	$41^\circ 17'$	315
80°	$40^\circ 42'$	251
75°	$39^\circ 46'$	206
70°	$38^\circ 29'$	153
65°	$36^\circ 53'$	125
50°	$30^\circ 29'$	058

The correspondence between the results given by experiment and those given by this calculation from Fresnel's formulæ was unexpected. The experiments were completed more than a month before any calculations were made, so that I had no idea at the time of making the experiments what would be the nature of the results given by calculation.

The differences between the two sets of values are hardly greater than the possible errors of observation. The differences at 65° and 50° are quite large, but might have been due to the lack of perfect uniformity in the luminous surfaces. On the whole, the agreement between the two sets of results indicates

strongly that in the case of uranium glass, at least, the phenomenon is one of simple refraction at the surface; but that the *WHOLE of the emitted light undergoes the refraction process.*

XI.

EXPERIMENTS UPON PLATINUM.

It is evident that no such comparisons as those just made for uranium glass could be made for the case of incandescent metals, unless, in the first place, the surface experimented upon could be assumed to be a perfectly definite, non-diffusing surface. The chief source of difficulty in the work upon platinum was to fulfill this condition.

It was found, after considerable work had been done upon platinum, that continual heating roughened the surface to a slight degree, and changed the amount of polarization. The results of several sets of laborious observations upon platinum were discarded altogether, because they were found to be erroneous from this cause. However, the change is so gradual that a well polished platinum surface may be heated to incandescence for several minutes without showing any perceptible change in character. The rapidity of the change could be delicately observed by viewing the surface at a large angle of incidence by means of the polarimeter. For a period of two or three minutes no change was perceptible in the equality of the images, but for much longer periods of heating the slow blistering of the surface began to be manifest in the disturbance of the equality of the images. Hence, in order to avoid this error, the surface of the platinum was carefully polished with rouge after every set of readings for a given angle.

A second slight source of error in the observations upon platinum was the lack of exact horizontality in the surface examined. The attempt was made to avoid this error by rotating the instrument through 90° according to the suggestion of Cornu. This brought the extraordinary image either above or below the ordinary; hence, when the angle of emergence was very large, the two images corresponded to points on the surface at a considerable distance from each other, as shown in the figure.



This introduced the likelihood of a much greater error than that due to a slight error in horizontality. The incandescent platinum was therefore rendered as nearly horizontal as possible by comparison with carefully levelled reference planes placed in the immediate vicinity. The adjustment could thus be easily made to within one degree.

In all of the following experiments, sheets of rolled platinum .06 mm. in thickness were heated to a white heat by means of a Bunsen burner, care being taken to prevent light from any other sources from vitiating the results. The observations are here given in full.

80°		70°		60°	
Left.	Right.	Left.	Right.	Left.	Right.
23.1	12.0	30.2	20.7	36.9	24.5
21.4	11.5	32.2	21.0	36.8	26.1
22.5	11.3	32.1	21.2	35.0	25.0
22.3	10.9	31.0	20.5	37.0	25.4
23.2	11.4	32.1	21.3	35.6	25.5
				35.5	25.4
22.5 11.49		31.52 20.94		36.05 25.2	
$2w = 34^\circ$		$2w = 52.4$		$2w = 61.25^\circ$	
$p = .829$		$p = .610$		$p = .481$	
50°		40°		30°	
Left.	Right.	Left.	Right.	Left.	Right.
40.0	29.7	45.6	35.0	47.6	38.0
39.1	29.0	45.5	33.3	47.5	36.3
41.2	29.3	44.4	34.2	47.2	36.2
39.4	28.5	43.8	34.6	47.8	37.1
41.0	29.6	44.4	34.5	48.0	36.0
40.14 29.2		44.74 34.3		47.62 36.72	
$2w = 69.34^\circ$		$2w = 79^\circ$		$2w = 84.34$	
$p = .349$		$p = .191$		$p = .099$	

XII.

APPLICATION OF CAUCHY'S FORMULÆ FOR METALLIC REFLECTION TO THE CASE OF PLATINUM.

Fresnel's formulæ for reflection rest upon the hypothesis that the time required in the process of reflection is infinitesimal in comparison with a wave period, and hence that the phase of vibration of the reflected ray is either the same as that of the incident ray, or else differs from it by the quantity π . It follows from this assumption that the reflected ray is plane polarized, if the incident ray is plane polarized.

When the reflection takes place at the boundary surface between air and a metal, experiment shows this assumption to be incorrect, and hence Fresnel's formulæ become inapplicable.

If the phenomenon here considered be due to reflection, the

laws for reflection which apply to the boundary surface between platinum and air are, of course, the laws to apply to the determination of the amounts of polarization which ought to be caused by a single refraction at this boundary.

The application of Fresnel's laws of vitreous reflection requires, as has been seen, the determination of but one constant, the index of refraction, or the ratio of the velocities of propagation of light in the two media. Cauchy extended these laws so as to cover the case of metallic reflection by introducing another constant which he calls the co-efficient of extinction. The constant corresponding to the index of refraction is, as in the case of transparent bodies, the tangent of the angle of maximum polarization. The co-efficient of extinction is a constant depending upon the opacity of the body, and is found from the ratio between the amplitudes, after reflection, of two equal beams polarized respectively perpendicular and parallel to the plane of incidence, and reflected at the angle of maximum polarization. This ratio is evidently the tangent of the azimuth of re-established plane polarization, when the incident beam is polarized in a plane making an angle of 45° with the plane of incidence, plane polarization being re-established after reflection by means of a quarter-wave plate or a Babinet compensator.

This angle may be determined by experiment. Thus the two constants of metallic reflection are, 1, the angle of maximum polarization, and 2, the azimuth of re-established plane polarization at this angle. According to the theory of Cauchy, these two constants being known, the intensity of a beam reflected at any angle may be calculated.

The complete explanation of Cauchy's theory and the deduction of Cauchy's formulæ were given by Eisenlohr in 1858. (see Pogg. Ann. 104, p. 368).

The final forms of the formulæ given by Eisenlohr are

$$K^2 = \tan(f - 45^\circ) \quad K'^2 = \tan(g - 45^\circ) \quad (1)$$

in which K^2 is the intensity of the reflected beam when the incident beam is polarized in the plane of incidence, K'^2 the intensity when the incident beam is polarized in the plane perpendicular to the plane of incidence, and f and g are variables given by the equations

$$\left. \begin{aligned} \cot f &= \cos(e + u) \sin\left(2 \arctan \frac{c \Theta}{\cos a}\right) \\ \cot g &= \cos(e - u) \sin\left(2 \arctan \frac{\cos a}{c \Theta}\right) \end{aligned} \right\} (2)$$

in which u and c are variables determined by the relations

$$\left. \begin{aligned} \cot (2u + e) &= \cot e \cos \left(2 \arctan \frac{\sin \alpha}{\Theta} \right) \\ c^2 &= \frac{\sin 2e}{\sin (2u + 2e)} \end{aligned} \right\} \quad (3)$$

in which α is the angle of incidence, and ε and ϑ are given by the final formulæ

$$\left. \begin{aligned} \sin 2e &= \tan^2 A \sin (4H - 2e) \\ \vartheta &= \sin A \sqrt{\frac{\sin 4H}{\sin (4H - 2e)}} \end{aligned} \right\} \quad (4)$$

in which A is the angle of maximum polarization called the "principle angle of incidence," and H is the azimuth of re-established plane polarization when the incident beam is polarized in azimuth 45° . H is called the "prime azimuth." The forms here given for ε and ϑ are due to Jochman (see Pogg. Ann. CXXXVI., p. 8561). These formulæ, first published by Cauchy in 1839, were shown by Jamin, by an elaborate series of measurements, to very closely represent the facts of reflection from metallic surfaces. The prime angles of incidence and the prime azimuths for all the common metals and for the different Fraunhofer lines were determined by Quincke in '74 (see Phil Mag. XLVII., p 221).

Now, in order to apply these formulæ to calculations similar to those which have already been made with Fresnel's formulæ upon uranium glass, it was necessary to assume, as before, that the whole of the light emitted had undergone refraction, and it was also necessary to know the two optical constants for platinum at the temperature of incandescence. These constants could not be determined. However, in a number of experiments made by W. R. Grove (see Phil. Mag. (4) 17, p. 177) upon the reflection of light from incandescent platinum, he was unable to detect any change in the reflecting properties of the platinum due to the fact of incandescence. Plane polarized light being reflected from the cold surface, the plane of polarization of the reflected beam was not affected by heating the platinum to the incandescent temperature. These experiments were not performed with delicate apparatus, yet they give reason to assume that the optical constants of platinum are not greatly altered by temperature.

Assuming, then, the values of A and H given by Quincke for the sodium line, the calculations of the amount of polarization in the emitted beam were made for all the angles of emergence

upon which experiments had been made. These calculations were made as follows :

Quinke's values for the D line are

$$\begin{array}{lcl} A = 77^{\circ} 8' & & \varepsilon = 64^{\circ} 22' \\ H = 32^{\circ} 46' & \text{Formulae (4) give} & \log \theta = .62276 \end{array}$$

Then for $a = 80^{\circ}$ formulæ (3) (2) and (1) give

$$K^2 = .9348 \qquad K'^2 = .4013$$

Assuming now the incident beam to have had an intensity unity, the emitted beam polarized in the plane of emergence would have an intensity

$$1 - K'^2 = .652$$

And the beam polarized in the plane perpendicular to the plane of emergence an intensity

$$1 - K'^2 = .5987$$

$$\text{Therefore the degree of polarization} = p = \frac{.5987 - .652}{.5987 + .652} = .834.$$

The complete results of the calculations for the platinum are as follows :

a	K^2	K'^2	$1-K^2$	$1-K'^2$	p
80°	.9348	.4013	.0652	.5987	.834
70°	.8757	.4044	.1243	.5957	.655
60°	.8234	.4813	.1766	.5187	.492
50°	.7782	.5483	.2218	.4517	.341
40°	.7409	.5981	.2591	.4019	.216
30°	.7115	.6330	.2885	.3670	.117

Considering the number of assumptions which have been made, the correspondence between these quantities and those given by experiment is altogether remarkable, and points with as much certainty as the work upon uranium glass to the conclusion that the phenomenon is simply one of refraction.

XIII.

DIFFERENCE IN COLOR OF IMAGES.

In the course of these observations upon platinum, another at first unaccountable phenomenon was noticed. At large angles of emergence the color in the two images was notably different. The feeble image, *i. e.*, the one corresponding to vibration perpendicular to the plane of emergence seemed for large angles to

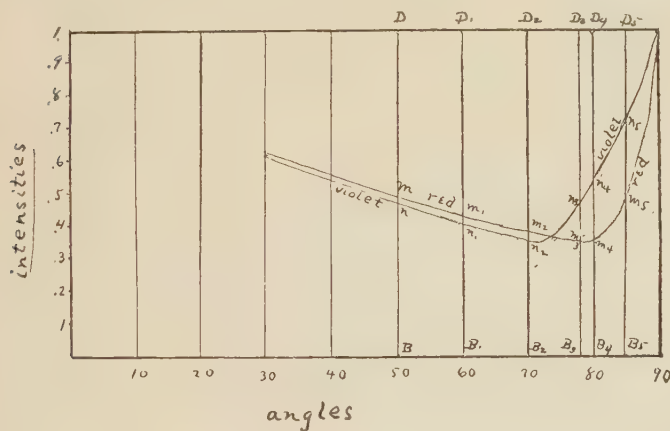
be deficient in red light, while the other image was abnormally red.

If we assume the phenomenon to be due to reflection and refraction, this appearance is readily explained by a reference to Quincke's values for the angles of maximum polarization for the different Fraunhofer lines.

This angle for the line C, Quincke gives as $78^{\circ} 28'$, and for the line G his value is $73^{\circ} 39'$.

Now if a is the amplitude of vibration in the reflected ray when the incident beam is polarized in the plane of incidence, and a' the amplitude when the plane of polarization of the incident beam is perpendicular to the plane of incidence the angle of maximum polarization will be reached when $\frac{a^2 - a'^2}{a^2 + a'^2}$ is a maximum, *i. e.*, when $\frac{a'}{a}$ is a minimum.

The experiments of Jamin show that this angle coincides, at least very nearly, with the angle for which a' is a minimum; a



conclusion which one would expect without the aid of experiment. Hence the angle $78^{\circ} 28'$ is that angle for which the component of the *reflected* vibration parallel to the plane of incidence is a minimum for the case of *red* light, and the component of the *emitted* vibration in the same plane is a maximum. On the other hand the angle of maximum emission of *violet* light in this plane occurred at $73^{\circ} 39'$. Accordingly it is evident that red light predominates in the beam emitted at the angle $78^{\circ} 28'$, and violet in the beam emitted at $73^{\circ} 39'$. The approximate

ratio between the two colors for any angle is shown in the diagram. It is evident that light emitted at any angle larger than 75° will be predominantly red. At the same time the shape of the curves accounts for the lack of any noticeable predominance of violet in the neighborhood of $73^\circ 39'$.

The figure shows the curves of intensities of the reflected components of vibration parallel to the plane of incidence as roughly plotted from the values given above. The lines $M D$, $M_1 D_1$, etc., represent the intensities of the emitted red vibrations in this plane for various angles; while the lines $N D$, $N_1 D_1$, etc., represent the intensities of the emitted violet vibrations for the same incidences. The lines $N M$, $N_1 M_1$, etc., are the measure of predominance of red over violet, or *vice versa*. The steepness of the curves at points corresponding to angles greater than the angle of maximum polarization and the lack of steepness at points corresponding to angles less than the angle of maximum polarization are evidently the cause of the marked predominance of red at large angles and the lack of noticeable predominance of violet at any angles. This characteristic of the curves follows from the fact that the points of maximum polarization correspond to very large angles.

XIV.

EXPERIMENTS UPON SILVER.

Owing to the great kindness of Mr. Herbert G. Torrey, Assayer of the U. S. Assay Office, I was next able to make a series of observations upon molten silver. These experiments were the most satisfactory of any which were made in the course of the research. All of the sources of error which had existed in preceding cases were here eliminated. The surface was perfectly defined, it was accurately horizontal, and there were no variations in intensity from point to point. The results of the experiments are given in full.

30°		35°		40°		45°	
Left.	Right.	Left.	Right.	Left.	Right.	Left.	Right.
46.0	36.0	44.5	33.4	43.3	31.7	42.0	30.5
47.2	36.0	45.0	33.7	42.8	32.5	42.0	31.0
46.5	35.0	44.5	34.0	43.3	32.0	41.1	31.3
46.3	35.0	45.0	34.0	43.5	32.0	42.5	30.5
46.5	35.5						
<u>46.5</u>	<u>35.5</u>	<u>44.75</u>	<u>33.8</u>	<u>43.72</u>	<u>32.05</u>	<u>41.9</u>	<u>30.8</u>
$2 w = .82^\circ$		$2 w = 78.55^\circ$		$2 w = 75.27^\circ$		$2 w = 72.7$	
$p = .139$		$p = .189$		$p = .254$		$p = .297$	

50°		55°		60°		65°	
Left.	Right.	Left.	Right.	Left.	Right.	Left.	Right.
40.6	28.5	38.0	27.0	34.7	24.1	32.0	21.0
40.0	29.0	38.3	27.5	35.0	24.0	32.5	21.5
40.0	29.8	37.5	27.1	34.0	24.1	32.6	21.0
40.5	29.1	37.5	26.3	35.4	23.8	32.5	20.5
40.0	28.8	38.0	26.5	35.7	24.0	32.5	21.5
				34.5	24.0		
<u>40.2 29.04</u>		<u>37.8 26.8</u>		<u>34.9 24.0</u>		<u>32.4 21.3</u>	
$2 w = 69.24^\circ$		$2 w = 64.6^\circ$		$2 w = 58.9^\circ$		$2 w = 53.7^\circ$	
$p = .354$		$p = .429$		$p = .517$		$p = .592$	

70°		75°		80°	
Left.	Right.	Left.	Right.	Left.	Right.
30.0	20.0	27.0	16.5	24.1	13.5
30.0	19.5	27.0	16.5	24.1	13.5
29.6	19.9	27.0	16.5	24.2	14.0
30.0	20.4	26.5	16.1	23.9	13.7
30.5	19.5	27.0	16.4	24.0	13.9
		26.5	15.5		
<u>30.0 19.9</u>		<u>26.8 16.25</u>		<u>24.1 13.7</u>	
$2 w = 49.9^\circ$		$2 w = 43.05$		$2 w = 37.8$	
$p = .644$		$p = .731$		$p = .789$	

All of the previous observations had been subject to errors of unknown magnitude, aside from the errors of observation; and the results, while agreeing very closely with the calculated values for some angles, differed from them by considerable amounts at others. For example, the agreement for platinum at 80° was very close, while at 70° the difference was as large as .045. Similarly for uranium glass, the difference at 50° and 65° was quite large. Hence I did not consider the results given by the experiments upon uranium glass and platinum altogether trustworthy as accurate quantitative measurements. The experiments upon silver, however, were free from all possible error so far as I was able to discover, except the observational error. Mention has already been made of the fact that Violle had previously made a number of determinations of the same general nature upon silver. His results do not agree very closely with those given above, being uniformly larger. I am altogether unable to account for the uniformity in the excess of his values over those given by these experiments. His results are here inserted for the sake of comparison.

Angle.	Violle.	Millikan.
30	.168	.139
50	.383	.354
60	.546	.517
65	.630	.592
70	.708	.644
75	.770	.731
80	.826	.789

The application of Cauchy's formulæ to silver was made in the same way as in the case of platinum. For polished silver Quincke gives for the D. line,

$$\begin{aligned} \text{Prime angle of Incidence, } A &= 72^\circ 10', \\ \text{Prime Azimuth, } H &= 41^\circ 40', \\ \text{Formulæ (4) give, } \left\{ \begin{array}{l} e = 82^\circ 34.3', \\ \log. \theta = .4480. \end{array} \right. \end{aligned}$$

Formulæ (3) (2) and (1) then give

a	K^2	K'^2	$1-K^2$	$1-K'^2$	p .
80	.9735	.8037	.0265	.1963	.762
75	.9606	.7628	.0394	.2372	.716
70	.9482	.7510	.0518	.2489	.655
65	.9361	.7540	.0639	.2460	.588
60	.9250	.7632	.0750	.2368	.519
55	.9136	.7740	.0864	.2260	.446
50	.9033	.7869	.0967	.2131	.376
45	.8937	.7985	.1063	.2015	.309
40	.8847	.8093	.1153	.1907	.246
35	.8767	.8187	.1233	.1813	.190
30	.8695	.8268	.1305	.1732	.140

It will be noticed that the agreement between these quantities p and those given by my experiments is closer than for either the platinum or the uranium glass; the largest difference being at 80° , where it amounts to .027.

XV.

EXPERIMENTS UPON GOLD AND IRON.

Through the kindness again of Mr. Torrey and the Superintendent of the Sub-Treasury, I was permitted to make observations upon a pot of molten gold; but accuracy of work was impossible on account of (1) the rapidity with which I was obliged

to work; (2) the lack of quiescence of the liquid surface; (3) the impossibility of excluding other light from the surface, and (4) the rapidity of oxidization of the molten gold. The results were therefore altogether untrustworthy as quantitative measurements. Hence no attempt was made to compare them with results given by Cauchy's formulae.

Molten iron was also made the subject of similar observations with equally unsatisfactory results.

XVI.

DISCUSSION OF RESULTS.

All of the comparisons made between experimental determinations and calculated values are condensed in the following tables.

URANIUM GLASS.			
Angle.	p. obs.	p. calc.	differ'nec.
87½	.358	.351	.007
85	.293	.315	-.022
80	.245	.251	-.006
75	.191	.206	-.015
70	.139	.153	-.014
65	.098	.125	-.027
50	.039	.058	-.019

PLATINUM.			
Angle.	p. obs.	p. calc.	differ'nec.
80	.829	.834	-.005
70	.610	.655	-.045
60	.481	.492	-.011
50	.349	.341	+.008
40	.191	.216	-.025
30	.099	.117	-.018

SILVER.			
Angle.	p. obs.	p. calc.	differ'nec.
80	.789	.762	+.027
75	.731	.716	+.015
70	.644	.655	-.011
65	.592	.588	+.004
60	.517	.519	-.002
55	.429	.446	-.017
50	.354	.376	-.022
45	.297	.309	-.012
40	.254	.246	+.008
35	.189	.190	-.001
30	.139	.140	-.001

In view of the general agreement between the observed and calculated values, and in view of the further fact of the coloration of the images at large angles, so beautifully accounted for by the reflection theory, it may be considered that the phenomenon of polarization of light by emission has thus been quantitatively proven to be a phenomenon of reflection and refraction.

It will be remembered that the apparently insuperable objection to the explanation which Arago offered was that that explanation attributed to all of the surface molecules the property of emitting natural light, and gave as the entire cause of the polarization, the refraction of light which works its way up from a certain depth beneath the surface.

The above calculations were all made upon the assumption

that *all* of the light emitted by the glowing body had undergone a refraction. Considering the closeness of agreement between the calculated and observed values, it is difficult to escape the conclusion that this assumption is correct, and that *no* particles whatever of the incandescent solid send out into the air natural light, save in the case in which the angle of emergence is zero. This simply means that all of the particles of the light emitting body, *including* the so-called surface layers, lie within the denser medium, and beneath the plane at which reflection and refraction take place. This relieves the refraction theory of the causes of the phenomenon of its greatest difficulty: viz., the difficulty of conceiving that, in the case of an exceedingly opaque body like platinum, the uppermost molecules send out but a very small proportion of the whole light emitted. If we follow the explanation of Arago and Verdet we are obliged by the results of this research to conclude that the emitted light originates almost entirely in molecules other than those of the uppermost layer. On the contrary it seems much more reasonable to assume that, in the case of such a body as platinum, the light emitted is due mainly to this topmost layer, but that the reflection process takes place entirely above the platinum.

Quincke has shown that when light from an external source is reflected at the surface of a metal the reflection does not take place in the geometrical plane between the two media, but rather takes place in the metal itself, the vibration penetrating for a certain depth into the denser medium. The converse is also doubtless true that the vibration originating in the metal is not reflected instantaneously at the surface of the rarer medium, but is reflected in the layer of air of finite thickness which borders upon the metal. Thus all light originating in the platinum, whether in the surface layer or the sub-surface layers, must undergo the process of reflection and refraction before it can emerge into the air.

Lastly, the calculated values were all obtained under the assumption that the optical constants of the metals are the same for high temperatures as for low; that is, that the reflecting properties of an incandescent metallic surface are precisely the same as the reflecting properties of a cold metallic surface. The closeness of agreement between the results given by this assumption and the facts as determined by experiment seems to warrant the conclusion, that the change in the optical properties of metals due to incandescence is *exceedingly slight*; a conclusion to which the somewhat inexact experiments of Grove upon the reflecting properties of incandescent platinum would also lead.

In conclusion, I will add that this investigation was suggested to me by Professor Rood, and I wish here to express my great indebtedness to him and to Professor Hallock, and also to Professor A. A. Michelson, of Chicago University, for aid furnished during its progress. I am also under obligations to Herbert G. Torrey, Esq., Assayer of the U. S. Assay Office, who most kindly placed at my disposal large masses of molten gold and silver.

R. A. MILLIKAN.

